Name: ____ Date: __________

Summer 2020 PreCalculus CP Packet
(for students entering PreCalculus CP in the Fall)

Pre Calculus presents the material which follows the study of Algebra and Plane Geometry and precedes the rigorous study of Calculus. It includes appropriate pre-calculus topics, trigonometry, and analytical geometry.

All students entering Pre Calculus CP must complete this math packet over the summer. It is due on **Friday, September 4, 2020**.

You will receive **2 grades** for this packet – one grade for completion and another grade for a summative assessment of the material. All problems should be completed with all work shown in a notebook. This will be checked on the first day of school.

In your PreCalculus class, you will be using a calculator often. Therefore, students are encouraged to buy their own **SCIENTIFIC CALCULATOR (Texas Instruments is suggested, TI-30XS).** Students are more efficient using a calculator with which they are familiar with.

For more practice or explanation of the skills in the packet, you may use the following as resources:

www.wolframalpha.com
www.khanacademy.com
www.ixl.com

If you lose your packet, there is a copy on the school department website.

Teachers: Ms. Paesano email: Delores.paesano@npsd.k12.ri.us
Ms. Cook email: Julie.Cook@npsd.k12.ri.us

Supplies:

- ✔ Notebook or binder with lined paper
- ✔ Pocket folder
- ✔ Pencils
- ✔ Colored pencils
- ✔ Scientific calculator
- ✔ An enthusiastic attitude!

SECTION 1.1 Lines in the plane - NOTES

Definition of the Slope of a Line

The **slope** m of the nonvertical line through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x}$$

where $x_1 \neq x_2$.

The Slope of a Line

1. A line with positive slope $(m > 0)$ *rises* from left to right.
2. A line with negative slope $(m < 0)$ *falls* from left to right.
3. A line with zero slope $(m = 0)$ is *horizontal*.
4. A line with undefined slope is *vertical*.

Point-Slope Form of the Equation of a Line

The **point-slope form** of the equation of the line that passes through the point (x_1, y_1) and has a slope of m is

$$y - y_1 = m(x - x_1).$$

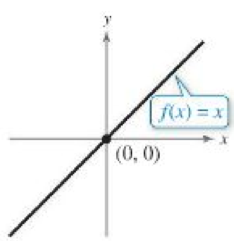
Library of Parent Functions: Linear Function

In the next section, you will be introduced to the precise meaning of the term *function*. The simplest type of function is the *parent linear function*

$$f(x) = x.$$

As its name implies, the graph of the parent linear function is a line. The basic characteristics of the parent linear function are summarized below and on the inside cover of this text. (Note that some of the terms below will be defined later in the text.)

Graph of $f(x) = x$
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Intercept: $(0, 0)$
Increasing



The function $f(x) = x$ is also referred to as the *identity function*. Later in this text, you will learn that the graph of the linear function $f(x) = mx + b$ is a line with slope m and y-intercept $(0, b)$. When $m = 0$, $f(x) = b$ is called a *constant function* and its graph is a horizontal line.

Slope-Intercept Form of the Equation of a Line

The graph of the equation

$$y = mx + b$$

is a line whose slope is m and whose y-intercept is $(0, b)$.

Summary of Equations of Lines

1. General form: $Ax + By + C = 0$
2. Vertical line: $x = a$
3. Horizontal line: $y = b$
4. Slope-intercept form: $y = mx + b$
5. Point-slope form: $y - y_1 = m(x - x_1)$

Parallel Lines

Two distinct nonvertical lines are **parallel** if and only if their slopes are equal. That is, $m_1 = m_2$.

Perpendicular Lines

Two nonvertical lines are **perpendicular** if and only if their slopes are negative reciprocals of each other. That is,

$$m_1 = -\frac{1}{m_2}.$$

EXERCISES

Vocabulary and Concept Check

1. Match each equation with its form.

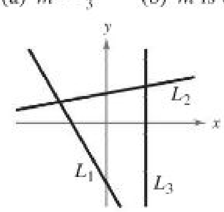
(a) $Ax + By + C = 0$	(i) vertical line
(b) $x = a$	(ii) slope-intercept form
(c) $y = b$	(iii) general form
(d) $y = mx + b$	(iv) point-slope form
(e) $y - y_1 = m(x - x_1)$	(v) horizontal line

In Exercises 2 and 3, fill in the blank.

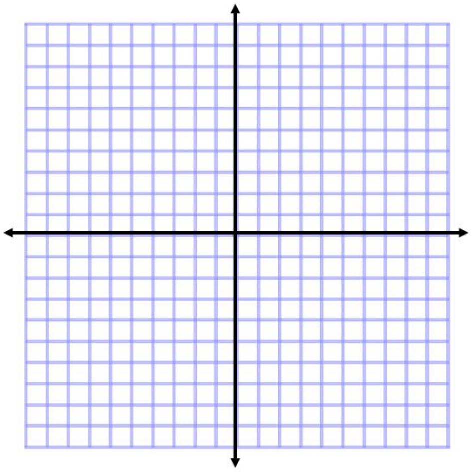
2. For a line, the ratio of the change in y to the change in x is called the _______ of the line.
3. Two lines are _______ if and only if their slopes are equal.
4. What is the relationship between two lines whose slopes are -3 and $\frac{1}{3}$?
5. What is the slope of a line that is perpendicular to the line represented by $x = 3$?
6. Give the coordinates of a point on the line whose equation in point-slope form is $y - (-1) = \frac{1}{4}(x - 8)$.

7. Identify the line that has the indicated slope.

(a) $m = \frac{2}{3}$ (b) m is undefined. (c) $m = -2$



8. Sketch the lines through the point with the indicated slopes on the same set of coordinates axes.



Point	*Slopes*			
(2, 3)	(a) 0	(b) 1	(c) 2	(d) -3

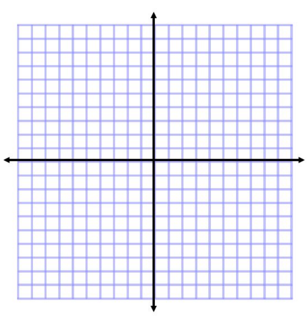
9. Find the slope of the line passing through the points (0, -10) and (-4, 0).

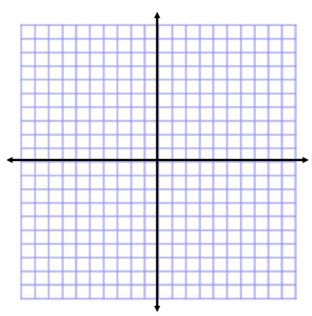
10. Find an equation of the line that passes through the given point and has the indicated slope. Sketch the line.

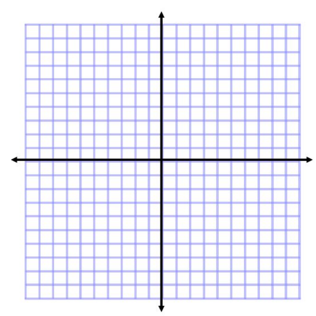
A. (0, -2), m = 3

B. (-10, 4), slope is undefined

C. (2, -8), m = 0







11. Determine the slope and y-intercept of the linear equation. Then describe the graph.

A. $2x - 3y = 9$

B. $x = -6$

12. Write an equation of the line that passes through the points (4, 3) and (-4, -4). Use the slope-intercept form.

13. Write the slope-intercept forms of the equations of the lines through the given point **(a)** parallel to the given line and **(b)** perpendicular to the given line.

(2, 1), $4x - 2y = 3$

14. Does every line have both an x-intercept and a y-intercept?

15. Does every line have an infinite number of lines that are parallel to it? Explain.

SECTION 1.2 Functions - NOTES

Definition of a Function

A **function** f from a set A to a set B is a relation that assigns to each element x in the set A exactly one element y in the set B. The set A is the **domain** (or set of inputs) of the function f, and the set B contains the **range** (or set of outputs).

Characteristics of a Function from Set A to Set B

1. Each element of A must be matched with an element of B.
2. Some elements of B may not be matched with any element of A.
3. Two or more elements of A may be matched with the same element of B.
4. An element of A (the domain) cannot be matched with two different elements of B.

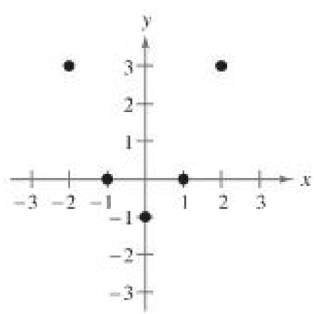
EXAMPLE 1 Testing for Functions

Determine whether each relation represents y as a function of x.

a.

Input, x	2	2	3	4	5
Output, y	11	10	8	5	1

b.



Solution

a. This table *does not* describe y as a function of x. The input value 2 is matched with two different y-values.

b. The graph *does* describe y as a function of x. Each input value is matched with exactly one output value.

EXAMPLE 3 Evaluating a Function

Let $g(x) = -x^2 + 4x + 1$. Find each value of the function.

a. $g(2)$ **b.** $g(t)$ **c.** $g(x + 2)$

Solution

a. Replacing x with 2 in $g(x) = -x^2 + 4x + 1$ yields the following.

$$g(2) = -(2)^2 + 4(2) + 1 = -4 + 8 + 1 = 5$$

b. Replacing x with t yields the following.

$$g(t) = -(t)^2 + 4(t) + 1 = -t^2 + 4t + 1$$

c. Replacing x with $x + 2$ yields the following.

$$g(x + 2) = -(x + 2)^2 + 4(x + 2) + 1 \quad \text{Substitute } x + 2 \text{ for } x.$$

$$= -(x^2 + 4x + 4) + 4x + 8 + 1 \quad \text{Multiply.}$$

$$= -x^2 - 4x - 4 + 4x + 8 + 1 \quad \text{Distributive Property}$$

$$= -x^2 + 5 \quad \text{Simplify.}$$

Library of Parent Functions: Absolute Value Function

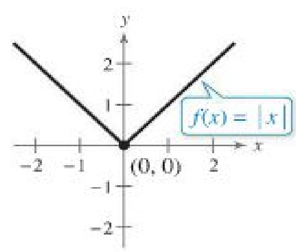
The *parent absolute value function* given by

$$f(x) = |x|$$

can be written as a piecewise-defined function. The basic characteristics of the parent absolute value function are summarized below and on the inside cover of this text.

Graph of $f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Intercept: $(0, 0)$
Decreasing on $(-\infty, 0)$
Increasing on $(0, \infty)$



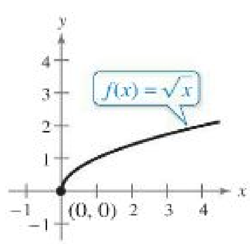
A function defined by two or more equations over a specified domain is called a **piecewise-defined function.**

Library of Parent Functions: Square Root Function

Radical functions arise from the use of rational exponents. The most common radical function is the *parent square root* function given by $f(x) = \sqrt{x}$. The basic characteristics of the parent square root function are summarized below and on the inside cover of this text.

Graph of $f(x) = \sqrt{x}$

Domain: $[0, \infty)$
Range: $[0, \infty)$
Intercept: $(0, 0)$
Increasing on $(0, \infty)$



Remark

Because the square root function is not defined for $x < 0$, you must be careful when analyzing the domains of complicated functions involving the square root symbol.

EXERCISES

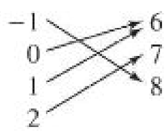
Vocabulary and Concept Check

In Exercises 1 and 2, fill in the blanks.

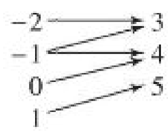
1. A relation that assigns to each element x from a set of inputs, or _______ , exactly one element y in a set of outputs, or _______ , is called a _______ .
2. For an equation that represents y as a function of x, the _______ variable is the set of all x in the domain, and the _______ variable is the set of all y in the range.
3. Can the ordered pairs (3, 0) and (3, 5) represent a function?
4. To find $g(x + 1)$, what do you substitute for x in the function $g(x) = 3x - 2$?
5. Does the domain of the function $f(x) = \sqrt{1 + x}$ include $x = -2$?
6. Is the domain of a piecewise-defined function *implied* or *explicitly described*?

Testing for Functions **In Exercises 7–10, does the relation describe a function? Explain your reasoning.**

7. *Domain* *Range*



8. *Domain* *Range*

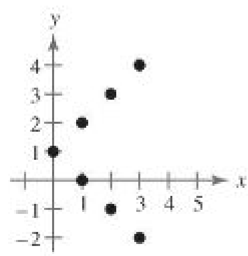


9. Determine whether the relation represents y as a function of x. Explain.

A.

Input, x	-3	-1	0	1	3
Output, y	-9	-1	0	1	-9

B.



10. Determine whether the equation represents y as a function of x.

A. $x^2 + y^2 = 4$ B. $y = \sqrt{x + 5}$ C. $y = |4 - x|$ D. $x = -7$

11. Evaluate the function at each specified value of the independent variable and simplify.

$f(x) = x^2 - 2x$ $A.\ f(2)$ $B.\ f(1.5)$ $C.\ f(x - 4)$

12. Find the values of x such that f(x) = 0.

A. $f(x) = 15 - 3x$ B. $f(x) = \frac{2x-3}{7}$

13. Find the domain of each function.

A. $f(x) = 1 - 2x^2$ B. $g(x) = \frac{4}{x}$ C. $h(x) = \frac{x+2}{\sqrt{x-10}}$

SECTION 1.3 Graphs of functions - NOTES

Vertical Line Test for Functions

A set of points in a coordinate plane is the graph of y as a function of x if and only if no vertical line intersects the graph at more than one point.

Definition of Relative Minimum and Relative Maximum

A function value $f(a)$ is called a **relative minimum** of f when there exists an interval (x_1, x_2) that contains a such that

$$x_1 < x < x_2 \quad \text{implies} \quad f(a) \le f(x).$$

A function value $f(a)$ is called a **relative maximum** of f when there exists an interval (x_1, x_2) that contains a such that

$$x_1 < x < x_2 \quad \text{implies} \quad f(a) \ge f(x).$$

EXAMPLE 8 Sketching a Piecewise-Defined Function

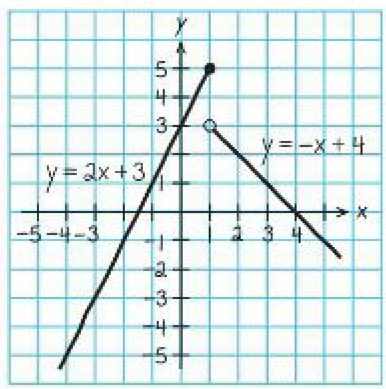
Sketch the graph of

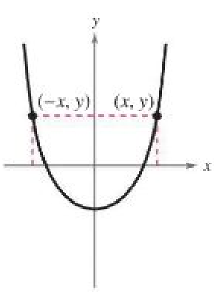
$$f(x) = \begin{cases} 2x + 3, & x \le 1 \\ -x + 4, & x > 1 \end{cases}$$

by hand.

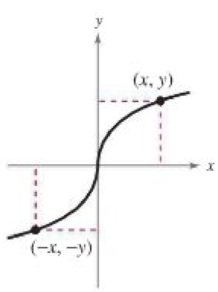
Solution

This piecewise-defined function is composed of two linear functions. At and to the left of $x = 1$, the graph is the line $y = 2x + 3$. To the right of $x = 1$, the graph is the line $y = -x + 4$, as shown in the figure. Notice that the point $(1, 5)$ is a solid dot and the point $(1, 3)$ is an open dot. This is because $f(1) = 5$.

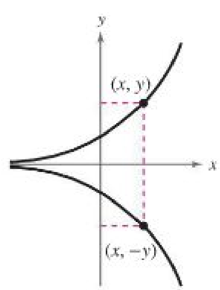




Symmetric to y-axis
Even function



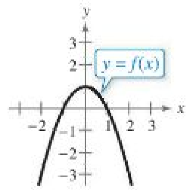
Symmetric to origin
Odd function



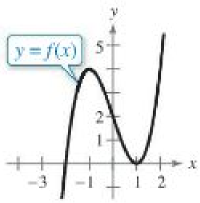
Symmetric to x-axis
Not a function

1. Use the graph of the function to find the **domain** and **range** of f. Then find $f(0)$.

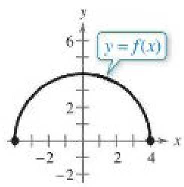
A.



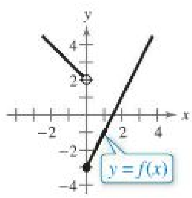
B.



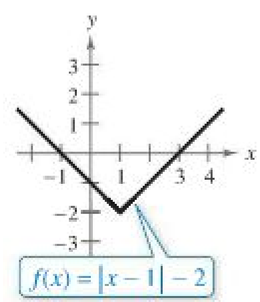
C.



D.



2. Use the graph of the function to answer the questions.



(a) **Determine the domain of the function.**

(b) **Determine the range of the function.**

(c) **Find the value(s) of x for which $f(x) = 0$.**

(d) **What are the values of x from part (c) referred to graphically?**

(e) **Find $f(0)$, if possible.**

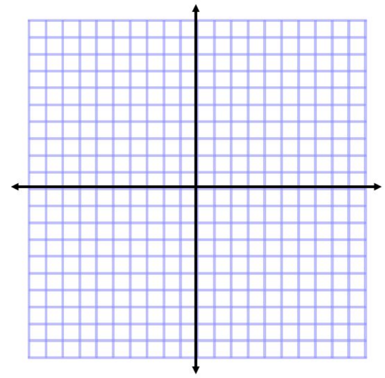
(f) **What is the value from part (e) referred to graphically?**

(g) **What is the value of f at $x = 1$? What are the coordinates of the point?**

(h) **What is the value of f at $x = -1$? What are the coordinates of the point?**

3. Sketch the graph of the piece-wise defined function.

$f(x) = \{x + 3, \quad x < 0 \; x^2 + 1, \quad x \geq 0$



Cumulative Review – Prior Knowledge

Operations with Rational Expressions In Exercises 93–96, perform the operation and simplify.

93. $12 - \frac{4}{x + 2}$

94. $\frac{3}{x^2 + x - 20} + \frac{2x}{x^2 + 4x - 5}$

95. $\frac{x^5}{2x^3 + 4x^2} \cdot \frac{4x + 8}{3x}$

96. $\frac{x + 7}{2(x - 9)} \div \frac{x - 7}{2(x - 9)}$

Identifying Polynomials In Exercises 117–122, determine whether the expression is a polynomial. If it is, write the polynomial in standard form.

117. $x + 20$

118. $3x - 10x^2 + 1$

119. $4x^2 + x^{-1} - 3$

120. $2x^2 - 2x^4 - x^3 + \sqrt{2}$

121. $\frac{x^2 + 3x + 4}{x^2 - 9}$

122. $\sqrt{x^2 + 7x + 6}$

Factoring Trinomials In Exercises 123–126, factor the trinomial.

123. $x^2 - 6x - 27$

124. $x^2 + 11x + 28$

125. $2x^2 + 11x - 40$

126. $3x^2 - 16x + 5$