

Common Core Standards Curriculum Map - Algebra II
Quarter One

Unit One - Linear Programming (8 days/4 blocks)

| <i>Common Core Standards and Content to Be Learned</i> | <i>Mathematical Practices and Essential Questions</i> | <i>Prior Learning, Current Learning and Future Learning</i> |
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| <p>Create equations that describe numbers or relationships [Equations using all available types of expressions, including simple root functions]. A-CED.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A-CED.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. <i>For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</i></p> <ul style="list-style-type: none"> • Interpret the verbal model to define the variables and write the objective function. • Represent constraints as equations or inequalities. • Graph systems of equations and/or inequalities on coordinate axes with labels and scales and determine a feasible region. • Identify important quantities in a practical situation and map their relationships. • Interpret the corner points to find the optimal solution. • Identify and interpret solutions as viable or non-viable options in a real-world context. | <p>SMP 1 Make sense of problems and persevere in solving them.</p> <p>SMP 4 Model with mathematics.</p> <p>SMP 5 Use appropriate tools strategically.</p> <p>How does the feasible region contribute to identifying solutions to a problem?</p> <p>How might the number of constraints affect possible solutions?</p> | <p><u>Prior Learning:</u> In grades 7, 8, and 9, students graphed linear equations and inequalities in two variables, solved 2 x 2 linear systems, graphed solutions to a system of inequalities in two variables, created a mathematical model from a verbal description, interpreted solutions, and determined appropriate domains.</p> <p><u>Current Learning:</u> Students create a mathematical model from a verbal description. They create equations and inequalities in one or two variables to represent relationships between quantities. They represent constraints as equations or inequalities. They graph systems of equations and/or inequalities on coordinate axes with labels and scales, and interpret solutions as viable or non-viable options in a modeling context.</p> <p><u>Future Learning:</u> Students will access prior knowledge when they determine the domain that reflects the context of a situation and when they determine optimal solutions. They will graph and interpret functions with more than one variable, identify bounded and unbounded regions, and determine when a problem has a unique solution, no solution, or infinitely many solutions. This knowledge transfers to calculus and college-level business courses.</p> |

Unit Two - Complex Numbers (6 days/3 blocks)

| <i>Common Core Standards and Content to Be Learned</i> | <i>Mathematical Practices and Essential Questions</i> | <i>Prior Learning, Current Learning and Future Learning</i> |
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| <p>Perform arithmetic operations with complex numbers. N-CN.1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real. N-CN.2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.</p> <p>Use complex numbers in polynomial identities and equations. N-CN.7. Solve quadratic equations with real coefficients that have complex solutions.</p> <ul style="list-style-type: none"> • Use the definition $i^2 = -1$ to simplify radicals. • Use the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. • Solve quadratic equations with real coefficients that have real and complex solutions. • Use the process of factoring and completing the square in quadratic functions to show real and complex zeros. | <p>SMP 6 Attend to precision.</p> <p>SMP 7 Look for and make use of structure.</p> <p>Why do imaginary numbers exist?</p> <p>When does a quadratic equation have imaginary solutions?</p> | <p>Prior Learning: In prior courses, students have operated with rational number systems and have understood the basic concepts of functions including linear and quadratic. In eighth grade, students learned that there are numbers that are not rational, and they approximated them by using rational numbers.</p> <p>Current Learning: Building on A-REI.4a and A-REI.4b, students are introduced to the relation $i^2 = -1$ and to the complex number system. Students solve quadratic equations and inequalities with complex solutions, and they perform operations with complex numbers.</p> <p>Future Learning: Students will interpret and model quadratic relationships between two quantities. They will use factoring, completing the square, and graphing to identify zeros, intercepts, and intervals where the functions are increasing and decreasing and positive and negative. Students will also find extreme values of functions. A key feature in graphing will be recognizing that not all zeros of functions are x-intercepts. In the fourth year course, students will apply the concepts of complex roots to higher-degree polynomial functions, and they will apply the Fundamental Theorem of Algebra. Students will use conjugates to find quotients of complex numbers and represent complex numbers and their operations geometrically on the complex plane.</p> |

Unit Three - Quadratic Functions (20 days/10 blocks)

| <p><i>Common Core Standards and Content to Be Learned</i></p> | <p><i>Mathematical Practices and Essential Questions</i></p> | <p><i>Prior Learning, Current Learning and Future Learning</i></p> |
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| <p>Use complex numbers in polynomial identities and equations. N-CN.7. Solve quadratic equations with real coefficients that have complex solutions.</p> <p>Create equations that describe numbers or relationships [Equations using all available types of expressions including simple root functions].</p> <p>A-CED.1. Create equations and inequalities in one variable and use them to solve problems. <i>Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</i></p> <p>A-CED.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</p> <p>Build a function that models a relationship between two quantities [For F.BF.1, 2 linear, exponential and quadratic].</p> <p>F-BF.3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (Just as an informal discussion as an introduction piece to the next unit.)</p> <p>Interpret functions that arise in applications in terms of the context [Emphasize selection of appropriate models].</p> <p>F-IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and</i></p> | <p>SMP 1 Make sense of problems and persevere in solving them.</p> <p>SMP 4 Model with mathematics.</p> <p>SMP 7 Look for and make use of structure.</p> <p>What are the connections between the algebraic solutions to a quadratic equation in one variable and the graph (including complex solutions)?</p> <p>What are the connections between the solutions to a quadratic equation in two variables, the table, and the symmetry of the graph?</p> <p>How do you identify when a real world problem is quadratic?</p> | <p>Prior Learning: In algebra 1, students used the structure of an expression to factor the difference of squares (A-SSE.2), and they factored quadratic expressions, found function zeros, completed the square, and found function maximums and minimums (A-SSE.3a and A-SSE.3b). Students completed the square to derive the quadratic formula and to transform quadratic equations. They solved quadratic equations by inspection, taking square roots, completing the square, using the quadratic formula, and factoring. Students learned to recognize complex solutions and write them in the form $a \pm bi$ for real numbers a and b (A-REI.4a and AREI4b). Previously in algebra 2, students were introduced to the definition $i^2 = -1$ and the complex number system, and they solved quadratic equations and inequalities with complex solutions; students also performed operations with complex numbers.</p> <p>Current Learning: Students interpret and model quadratic relationships between two quantities. They use factoring, completing the square, and graphing to identify zeros, intercepts, and intervals where the functions are increasing and decreasing and positive and negative, and they find extreme values. A key feature in graphing is recognizing that not all zeros of functions are x-intercepts.</p> <p>Future Learning: In the fourth-year course, students will apply the concepts of complex roots to higher-degree polynomial functions and apply the Fundamental Theorem of Algebra. They will use conjugates to find quotients of complex numbers and represent complex numbers and their operations geometrically on the complex plane.</p> |

minimums; symmetries; end behavior; and periodicity. ★

F-IF.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* ★

Analyze functions using different representations [Focus on using key features to guide the selection of appropriate models of the function].

F-IF.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★

F-IF.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

F-IF.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

- Create equations and inequalities in one variable and use them to solve quadratic problems.
- Create and graph equations and inequalities in two variables to represent quadratic relationships between quantities.
- Interpret key features and sketch graphs of quadratic relationships from verbal descriptions including zeros, intercepts, intervals of increase and decrease, intervals of positive and negative values, extreme values, symmetries, and end behaviors.
- Use the domain to determine the reasonableness of solutions to quadratic applications.
- Compare the properties of two quadratic functions, each represented in a different way (i.e., one in algebraic form and one in table form).

Unit Four - Functions Overview (6 days/3 blocks)

| <p><i>Common Core Standards and Content to Be Learned</i></p> | <p><i>Mathematical Practices and Essential Questions</i></p> | <p><i>Prior Learning, Current Learning and Future Learning</i></p> |
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| <p>Build a function that models a relationship between two quantities [For F-BF.1,2 linear, exponential and quadratic]. F-BF.3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.</p> <p>Interpret functions that arise in applications in terms of the context <i>[Emphasize selection of appropriate models].</i> F-IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</i> ★ F-IF.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. <i>For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.</i> ★</p> <p>Analyze functions using different representations <i>[Focus on key features to guide selection of appropriate models of the function].</i> F-IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★</p> | <p>SMP 1 Makes sense of problems and persevere in solving them.</p> <p>SMP 4 Model with mathematics.</p> <p>SMP 7 Look for and make use of structure.</p> <p>What are the effects on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative)?</p> <p>How would you compare and contrast the domain, range, rate, and end behavior of the following parent functions and transformations of them: $f(x) = x^2$, $f(x) = x$, $f(x) = \sqrt{x}$, $f(x) = x^3$, $f(x) = x^3$, piecewise, and step functions?</p> <p>What are the key features of any function such as: minimums, maximums, intercepts, and increasing and decreasing intervals?</p> | <p>Prior Learning: In grade 8, students interpreted and constructed linear functions. In algebra 1, students learned the concepts of a function and the use of function notation. They have interpreted linear, exponential, and quadratic functions in applications or in terms of a context. They have analyzed linear, exponential, quadratic, absolute value, step, and piecewise functions using a graphical representation.</p> <p>Current Learning: Students graph and identify key features of the parent functions $f(x) = x^2$, $f(x) = x$, $f(x) = \sqrt{x}$, $f(x) = x^3$, $f(x) = 3x$. They also graph piece-wise and step functions.</p> <p>Future Learning: In the fourth-year course, students will analyze logarithmic and trigonometric functions using different representations.</p> |

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

- Graph and identify key features of the parent functions $f(x) = x^2$, $f(x) = |x|$, $f(x) = \sqrt{x}$, $f(x) = x^3$, $f(x) = 3x$.
- Explore transformations of selected functions.
- Graph piece-wise and step functions.
- Given a graph, determine domain, range, intercepts, end behavior, minimums, maximums, symmetries, and intervals where the function is positive, negative, increasing, decreasing, and/or constant.

Quarter Two

Unit Five - Polynomial Functions Beyond Quadratics (25 days/12-13 blocks)

| <i>Common Core Standards and Content to Be Learned</i> | <i>Mathematical Practices and Essential Questions</i> | <i>Prior Learning, Current Learning and Future Learning</i> |
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| <p>Create equations that describe numbers or relationships [Equations using all available types of expressions including simple root functions]. A-CED.1. Create equations and inequalities in one variable and use them to solve problems. <i>Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</i> A-CED.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</p> <p>Interpret the structure of expressions [Polynomial and rational]. A-SSE.1. Interpret expressions that represent a quantity in terms of its context. ★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. <i>For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.</i> A-SSE.2. Use the structure of an expression to identify ways to rewrite it. <i>For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.</i></p> <p>Perform arithmetic operations on polynomials [Beyond quadratic]. A-APR.1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.</p> | <p>SMP 1 Make sense of problems and persevere in solving them.</p> <p>SMP 4 Model with mathematics.</p> <p>SMP 7 Look for and make use of structure.</p> <p>How does the degree of a polynomial function affect its behavior and why?</p> <p>What are polynomial functions and how do we graph them?</p> <p>How is polynomial division connected to the graph of the function?</p> <p>How do you identify when a real world problem should be modeled by a polynomial function?</p> | <p>Prior Learning: Students have recognized expressions as linear, exponential, or quadratic and written equivalent quadratic expressions. They have represented linear, exponential, quadratic, absolute value, step and piecewise functions graphically. Students solved quadratic equations including those with complex solutions by factoring, extracting the root, completing the square, and the quadratic formula. They have performed arithmetic operations on linear and quadratic polynomials.</p> <p>Current Learning: Students add, subtract, multiply, and divide polynomial expressions; factor sum and difference of cubes; classify polynomials by degree and number of terms; and describe and model relationships involving polynomial identities and use them to solve problems. Students represent and solve polynomial equations algebraically and graphically. They know and make use of the Remainder Theorem. They graph polynomial functions and identify intercepts and intervals where the function is increasing, decreasing, positive or negative. Students also find relative maximum or minimums, symmetries, and end behavior. They identify and understand zeros and multiplicity of zeros in the related graph. Students use polynomial models to solve real-world problems.</p> <p>Future Learning: In the fourth course, students will find and use the conjugate of a complex number. They will represent complex numbers in rectangular and polar form and represent the operations geometrically.</p> |

Understand the relationship between zeros and factors of polynomials.

A-APR.2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a , the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

A-APR.3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Use polynomial identities to solve problems.

A-APR.4. Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.*

Represent and solve equations and inequalities graphically [Combine polynomial, rational, radical, absolute value, and exponential functions].

A-REI.11. Explain why the x -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ★

Build a function that models a relationship between two quantities [For F.BF.1, 2, linear, exponential and quadratic].

F-BF.1. Write a function that describes a relationship between two quantities. ★

b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

Interpret functions that arise in applications in terms of

context [Emphasize selection of appropriate models].

F-IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* ★

Analyze functions using different representations [Focus on key features to guide selection of appropriate models of the function].

F-IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

- Classify polynomials by degree and terms.
- Perform arithmetic operations on polynomial expressions including addition, subtraction, multiplication, and division (long and synthetic).
- Compare representations of different polynomial functions.
 - o Interpret key features of graphs and tables including intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximum or minimums; symmetries; and end behavior.
 - o Identify and understand zeros and multiplicity of zeros in the related graph.
- Write polynomial expressions in standard and factored form.
 - o Factor sum/difference of cubes.
- Use polynomial models to solve real-world problems.
- Sketch the graphs of polynomial functions.
- Understand and apply the Remainder Theorem.

Unit Six - Rational Functions (15 days/7-8 blocks)

| <p><i>Common Core Standards and Content to Be Learned</i></p> | <p><i>Mathematical Practices and Essential Questions</i></p> | <p><i>Prior Learning, Current Learning and Future Learning</i></p> |
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| <p>Rewrite rational expressions [Linear and quadratic denominators]. A-APR.6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. A-APR.7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.</p> <p>Understand solving equations as a process of reasoning and explain the reasoning [Simple radical and rational]. A-REI.2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.</p> <p>Build new functions from existing functions [Include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types]. F-BF.1. Write a function that describes a relationship between two quantities. ★ b. Combine standard function types using arithmetic operations. <i>For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.</i> F-BF.3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the</p> | <p>SMP 2 Reason abstractly and quantitatively</p> <p>SMP 4 Model with mathematics.</p> <p>SMP 7 Look for and make use of structure</p> <p>How does changing the values of h and k affect the graph of $h(x)=a/(x-h) +k$?</p> <p>What causes horizontal asymptotes, vertical asymptotes, and removable discontinuities to occur in the graphs of rational functions?</p> <p>How do you use operations of rational numbers to perform operations on rational expressions?</p> <p>What is the domain of a word problem represented by a rational function?</p> <p>How do you identify when a real</p> | <p>Prior Learning: Students have simplified polynomial expressions. They have solved linear and polynomial (including quadratic) equations. Students graphed polynomial functions and identified domain, range, and intercepts. They built new functions from existing ones and graphed transformations of select parent functions.</p> <p>Current Learning: Students rewrite rational expressions in different forms, where the denominator is linear or quadratic. Students solve rational equations; they explain their reasoning with regard to rational functions and build new rational functions. As appropriate, they graph rational functions indicating domain, horizontal and vertical asymptotes, and removable discontinuities.</p> <p>Future Learning: Students will apply transformations to rational functions. They will solve systems involving non-linear equations.</p> |

Adapted from the Charles A. Dana Center work with SORICO 2012

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| <p>effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.</p> <p>Create equations that describe numbers of relationships [Equations using all available types of expressions, including simple root functions]. A-CED.1. Create equations and inequalities in one variable and use them to solve problems. <i>Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</i></p> <p>A-CED.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</p> <p>Interpret functions that arise in applications in terms of context [Emphasize selection of appropriate models].</p> <p>F-IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</i> ★</p> <p>F-IF.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. <i>For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.</i> ★</p> <p>F-IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★</p> <p>d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.</p> <p>F-IF.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</i></p> | <p>world problem should be modeled by a rational function?</p> | |
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Understand solving equations as a process of reasoning and explain the reasoning [*Simple ~~radical and rational~~*].

A-REI.2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Represent and solve equations and inequalities graphically [*Combine polynomial, rational, ~~radical~~, absolute value, and exponential functions*].

A-REI.11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ★

- Rewrite rational expressions in different forms, where the denominator is linear or quadratic.
- Solve rational equations.
- Build new rational functions from previously learned rational functions.
- Determine the domain of rational functions.
- Determine horizontal and vertical asymptotes using their definitions.
- Identify any removable discontinuities.
- Graph rational functions, indicating intercepts.

Quarter Three

Unit Seven - Radical Functions (8 days/4 blocks)

| <i>Common Core Standards and Content to Be Learned</i> | <i>Mathematical Practices and Essential Questions</i> | <i>Prior Learning, Current Learning and Future Learning</i> |
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| <p>Interpret functions that arise in applications in terms of context [Emphasize selection of appropriate models].</p> <p>F-IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</i> ★</p> <p>F-IF.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. <i>For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.</i> ★</p> <p>F-IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★</p> <p style="padding-left: 20px;">b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</p> <ul style="list-style-type: none"> ● Solve radical equations. ● Determine if an equation has any extraneous solutions. ● Determine the domain of radical functions. ● Graph radical functions, indicating intercepts. ● Connect the graph of radical functions to the solutions o radical equations. | <p>SMP 1 Make sense of problems and persevere in solving them.</p> <p>SMP 4 Model with mathematics.</p> <p>SMP 7 Look for and make use of structure.</p> <p>Why do radical equations sometimes have extraneous solutions?</p> <p>How can you determine if a solution to a radical equation is extraneous?</p> <p>How do you determine the domain of a rational function and how does this connect to the solutions of a radical equation?</p> <p>How can you use your knowledge of function transformations to graph a radical function?</p> | |

Unit Eight - Exponential Functions (16 days/8 blocks)

| <i>Common Core Standards and Content to Be Learned</i> | <i>Mathematical Practices and Essential Questions</i> | <i>Prior Learning, Current Learning and Future Learning</i> |
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| <p>Create equations that describe numbers or relationships [Equations using all available types of expressions, including simple root functions]. A-CED.1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. A-CED.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A-CED.4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance R.</p> <p>Build new functions from existing functions. F-BF.5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.</p> <p>Interpret functions that arise in application in terms of context [Emphasize selection of appropriate models]. F-IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★ F-IF.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. ★</p> | <p>SMP 2 Reason abstractly and quantitatively.</p> <p>SMP 4 Model with mathematics.</p> <p>SMP 5 Use appropriate tools strategically.</p> <p>SMP 6 Attend to precision.</p> <p>How is the process of solving equations and rearranging formulas for a quantity of interest similar or different?</p> <p>How do you identify when a real world problem should be modeled by an exponential function?</p> <p>By looking at the equation for an exponential model, how can you determine whether it is a growth or decay model and why?</p> <p>What are the key graphical features of an exponential function?</p> <p>Why does the parent graph of an</p> | <p>Prior Learning: In Algebra 1, students learned to create linear equations. They were also introduced to quadratic and exponential equations. In the first unit of Algebra 2, students learned to create equations, including quadratic. Grade 8 students graphed linear functions and learned to identify domain and intercepts. In Algebra 1, they were introduced to simple exponential functions and their graphs. In Unit 2.2 of Algebra 2, students learned to create equations that describe relationships.</p> <p>Current Learning: Students learn to create exponential equations and inequalities with one and two or more variables. They also learn to represent equations and inequalities on the coordinate grid with proper labels and scales. Students learn to understand and interpret solutions as viable or nonviable. Students also learn to rearrange formulas for a quantity of interest. Students learn to represent and solve exponential equations. They learn to use technology in the process of solving exponential functions. Students learn to differentiate between exponential growth and decay. They learn to relate the domain of a function to its graph.</p> <p>Future Learning: Students will use exponential equations in Precalculus when learning about inverse functions. They will also use this knowledge when working with logarithmic functions. In Pre-calculus, students will extend exponential functions to include relative extremes. This also extends to the Pre-calculus topic of concavity. Students in Calculus will determine the area under the curve and infinite rectangles.</p> |

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| <p>Analyze functions using different representations [Focus on using key features to guide selection of appropriate models for the function].</p> <p>F-IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★</p> <p>e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.</p> <p>F-IF.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.</p> <p>b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.</p> <p>Write expressions in equivalent forms to solve problems.</p> <p>A-SSE.4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments. ★</p> <ul style="list-style-type: none"> • Create exponential equations in one variable. • Create exponential equations in two or more variables. • Use equations to solve real world problems. • Represent relationships and constraints between quantities. • Rearrange formulas to solve for quantity of interest. • Graph and solve exponential functions. • Use technology to find solutions to exponential functions. • Identify and interpret key features of exponential functions. • Graphically represent exponential growth and decay. • Determine the range of exponential functions, and relate the range to the graph of the function. | <p>exponential function only exist for functional values greater than zero? How and why does the range change when you shift this model up or down?</p> <p>How is the domain of an exponential function related to its graph?</p> <p>What is the importance of the use of technology in solving exponential equations and inequalities?</p> | |
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Unit Nine - Logarithmic Functions (16 days/8 blocks)

| <p><i>Common Core Standards and Content to Be Learned</i></p> | <p><i>Mathematical Practices and Essential Questions</i></p> | <p><i>Prior Learning, Current Learning and Future Learning</i></p> |
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| <p>Build new functions from existing functions. F-BF.5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. Interpret functions that arise in application in terms of context [Emphasize selection of appropriate models]. F-IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★ F-IF.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. ★</p> <p>Analyze functions using different representations [Focus on using key features to guide selection of appropriate models for the function]. F-IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★ e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.</p> <p>Construct and compare linear, quadratic and exponential models and solve problems [Logarithms as solutions for exponentials].</p> | <p>SMP 2 Reason abstractly and quantitatively.</p> <p>SMP 4 Model with mathematics.</p> <p>SMP 5 Use appropriate tools strategically.</p> <p>SMP 7 Look for and make use of structure.</p> <p>SMP 8 Look for and express regularity in repeated reasoning.</p> <p>What is a method for solving logarithmic equations?</p> <p>What are the characteristics of a logarithmic function and how do these characteristics relate to exponential functions?</p> <p>How do you use the characteristics of inverse functions to graph logarithmic functions?</p> <p>What is the process for solving logarithmic equations?</p> | <p>Prior Learning: In Algebra 1, students were introduced to exponential functions and their graphs. They also learned to find inverse functions. Early in Algebra 2, students learned to graph exponential functions. In Algebra 1, students were introduced to exponential functions and their graphs. They also learned to find inverse functions. Early in Algebra 2, students learned to graph exponential functions. They also learned the relationship between exponential and logarithmic functions. Students learned to identify the domain of a logarithmic function and to convert to and from logarithmic equations to exponential equations. Students also learned about the base e.</p> <p>Current Learning: In Algebra 2, students learn the relationship between exponential and logarithmic functions. They also learn to identify the domain of a logarithmic function. Students convert to and from logarithmic equations to exponential equations. Students also learn about the base e. Students learn how the parameters affect the graph of a logarithmic function. They understand the relationship of the domain and its function, and they understand key features of a graph. Students use this understanding to graph logarithmic functions by hand.</p> <p>Future Learning: Algebra 2 students will learn about intercepts and end behavior of logarithmic functions. They will also learn about change of base formula. In Pre-calculus, students will graph logarithmic functions. They will also use the concept of inverse functions and apply it to the trigonometric functions. In Pre-calculus, students will graph logarithmic functions. In Calculus, they will use these concepts to find the derivatives and integrals of functions.</p> |

F-LE.4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a , c , and d are numbers and the base b is 2, 10, or e ; evaluate the logarithm using technology.

- Create equations in one and two or more variables.
- Graph equations on coordinate axes.
- Use logarithms to solve for an unknown exponent.
- Find inverse functions.
- Identify the domain of inverse functions and the graphs of inverse functions.
- Evaluate logarithms using technology.
- Interpret solutions as viable or nonviable.
- Use technology to graph logarithmic functions.
- Make table of values for the graph of a logarithmic functions.
- Determine the relationship of the domain and graph of a logarithmic function.
- Sketch a graph of a logarithmic function, given key features verbally.
- Interpret key features of graphs of logarithmic functions in terms of quantity.
- Graph logarithmic functions and identify end behavior and intercepts.

How can you use the graph of a logarithmic function to verify its solution?

What are the characteristics of logarithmic functions and how do they relate to the characteristics of exponential functions?

How does changing the parameters of the parent function affect the graph?

Quarter Four

Unit Ten - Trigonometry - The Unit Circle, Radian Measure, Angles of Rotation (15 days/7-8 blocks)

| <i>Common Core Standards and Content to Be Learned</i> | <i>Mathematical Practices and Essential Questions</i> | <i>Prior Learning, Current Learning and Future Learning</i> |
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| <p>Extend the domain of trigonometric functions using the unit circle. F-TF.1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. F-TF.2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.</p> <ul style="list-style-type: none"> • Recognize the radian measure of an angle as the arc length on the unit circle. • Determine the connection between the unit circle and radian measures. • Prove the Pythagorean trigonometric identity. • Find trigonometric angle measures using trigonometric identities. | <p>SMP 2 Reason abstractly and quantitatively.</p> <p>SMP 5 Use appropriate tools strategically.</p> <p>SMP 7 Look for and make use of structure.</p> <p>SMP 8 Look for and express regularity in repeated reasoning.</p> <p>What is the relationship between the radian measure of an angle and the unit circle?</p> <p>How can the sine, cosine, and tangent functions be defined using the unit circle?</p> <p>How can you use a given value of a trigonometric function to determine the values of other functions?</p> <p>What method(s) are used to determine the Pythagorean trigonometric identity?</p> | <p><u>Prior Learning:</u> In grade 8, students were introduced to the Pythagorean Theorem and its application. Geometry students used right-angle trigonometry and the Laws of Sines and Cosines. Geometry students also derived the equation of a circle.</p> <p><u>Current Learning:</u> Students determine the relationship between radian measures and arc length on the unit circle. They also identify the trigonometric functions on the unit circle and their measures. Students determine the Pythagorean trigonometric identity and use it to determine the value of other functions.</p> <p><u>Future Learning:</u> Precalculus students will continue to determine trigonometric values on the unit circle. They will also continue to use the unit circle to explain symmetry and periodicity of trigonometric functions. Precalculus students will graph trigonometric functions, identifying zeros and asymptotes. They will also use inverse functions to solve trigonometric equations.</p> |

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| | What is the relationship between the trigonometric angle measures and their identities? | |
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Unit Eleven - Trigonometric Functions - Sine and Cosine (10 days/5 blocks)

| <p><i>Common Core Standards and Content to Be Learned</i></p> | <p><i>Mathematical Practices and Essential Questions</i></p> | <p><i>Prior Learning, Current Learning and Future Learning</i></p> |
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| <p>Build new functions from existing functions [Include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types]. F-BF.3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.</p> <p>Model periodic phenomena with trigonometric functions. F-TF.5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. ★</p> | <p>SMP 5 Use appropriate tools strategically.</p> <p>SMP 7 Look for and make use of structure.</p> <p>How do you identify when a real world problem should be modeled by a trigonometric function?</p> <p>What are the key criteria to consider when modeling a trigonometric function and how do you integrate these into the equation and the graph?</p> <p>How do you relate the properties of function transformations previously learned to the key vocabulary of trigonometric functions?</p> | |

Unit Twelve - Basic and Pythagorean Trigonometric Identities (4 days/2 blocks)

| <p><i>Common Core Standards and Content to Be Learned</i></p> | <p><i>Mathematical Practices and Essential Questions</i></p> | <p><i>Prior Learning, Current Learning and Future Learning</i></p> |
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| <p>F-TF.8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.</p> | <p>SMP 6 Attend to precision.</p> <p>How do you begin to prove a trigonometric identity and how can this be done in different ways?</p> <p>What strategies can be used to prove a trigonometric identity?</p> | |

Unit Thirteen - Statistics - Experiments, Surveys, and Observational Studies (5 days/2-3 blocks)

| <p><i>Common Core Standards and Content to Be Learned</i></p> | <p><i>Mathematical Practices and Essential Questions</i></p> | <p><i>Prior Learning, Current Learning and Future Learning</i></p> |
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| <p>Understand and evaluate random processes underlying statistical experiments. S-IC.1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population. S-IC.2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. <i>For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?</i></p> <p>Make inferences and justify conclusions from sample surveys, experiments, and observational studies. S-IC.3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. S-IC.4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. S-IC.5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. S-IC.6. Evaluate reports based on data.</p> | <p>SMP 1 Make sense of problems and persevere in solving them.</p> <p>SMP 2 Reason abstractly and quantitatively.</p> <p>What is the purpose of creating a statistical model?</p> <p>How do you decide if a specific model is consistent with the results from a given data-generating process?</p> <p>How do you create a simulation to model an event?</p> <p>How do you use sample surveys to estimate a population mean or proportion and what other factors must you consider when you do this?</p> | |

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